# VP160 Recitation Class

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# Angular Momentum

## Particle

$$\overline{L} = \overline{r} \times \overline{P}$$

$$\overline{L} = I \cdot \overline{\omega}$$

$$\overline{\tau} = \overline{r} \times \overline{F}$$

$$\overline{\tau} = \frac{d\overline{L}}{dt}$$

## Rigid Body

$$\overline{L} = \sum_{i=1}^{N} m_i r_i \times (\omega \times r_i)$$

$$I = \sum_{i=1}^{n} m_i r_i^2$$

$$\overline{L} = I \cdot \overline{\omega}$$

## Tensor Representation

$$\left[\bar{I}_{\alpha'\beta'}\right]_{\alpha',\beta'=x',y',z'} = \begin{bmatrix} \sum_{i=1}^{N} m_i(y_i'^2+z_i'^2) & -\sum_{i=1}^{N} m_i x_i' y_i' & -\sum_{i=1}^{N} m_i x_i' z_i' \\ -\sum_{i=1}^{N} m_i y_i' x_i' & \sum_{i=1}^{N} m_i(x_i'^2+y_i'^2) & -\sum_{i=1}^{N} m_i y_i' z_i' \\ -\sum_{i=1}^{N} m_i x_i' z_i' & -\sum_{i=1}^{N} m_i z_i' y_i' & \sum_{i=1}^{N} m_i(x_i'^2+y_i'^2) \end{bmatrix}$$

## Conservation of Angular Momentum

If the sum of all external torques on the system is equal to zero, then the total angular momentum of the system is constant (planar motion). The total angular momentum of a system can only be changed by external torques.

## Energy in Rotation

$$E_k = \frac{1}{2} I_c \omega^2 + \frac{1}{2} m v_c^2$$

## Parallel-axis Theorem(Steiner's Theorem)

$$I_O = I_C + m\overline{OC}^2$$

## Perpendicular-axis theorem

$$I_x + I_y = I_z$$

## Rolling without Slipping

$$\omega r_c = v_c$$

## Rigid Body Dynamics

1. Newton’s Laws
2. Dynamics Laws for rotational motion
3. Conservation of energy
4. Conservation of momentum
5. Conservation of angular momentum

## Exercise 1

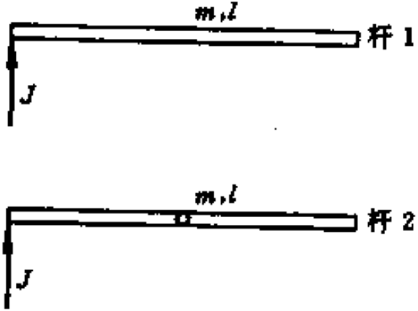
A cylindrical shaped pen was spinning around its axis at a constant angular velocity *omega* in the universe(no external force). At one moment, an instantaneous force was applied on one end of the pen, perpendicular to the axis. Discuss the motion of the pen later.

# Exercise 2

1. Find the principle moment of inertia $I$ of a stick with length $l$ and mass $m$;
2. Find the principle moment of inertia $I$ of a circle with radius $r$ and mass $m$;
3. Find the principle moment of inertia $I$ of a disk with radius $r$ and mass $m$;
4. Find the principle moment of inertia $I$ of a ball with radius $r$ and mass $m$;
5. Find the principle moment of inertia $I$ of a square with length of side $a$ and mass $m$.
6. Find the moment of inertia $I$ of a stick around one of its ends with length $l$ and mass $m$;
7. Find the moment of inertia $I$ of a disk around one of its diameter with radius $r$ and mass $m$;

# Exercise 3

Body 1 consists of one stick with mass $m$ and length $l$. Body 2 consists of two sticks with mass $m/2$ and length $l/2$. The two sticks are connected with each other with a hinge. Apply an impulse $J$ to these two bodies, as shown in the figure. Find $E_1/E_2$, where $E_1$ and $E_2$ are the energy of the two bodies respectively.



## Exercise 4

A ball is spinning anticlockwise with angular velocity $\omega$. The speed of the ball at this moment is $v$, pointing right. The friction coefficient of the ground is $\mu$, which is strong enough. Discuss the motion of the ball after this moment. What if a circle? What if a disk?

## Exercise 5

A ball is placed on a cant with dip angle $\theta$. The friction coefficient is $\mu$, which is high enough. Release the ball and discuss the motion afterwards. What if a circle? What if a disk? What if a regular pentagon?

## Exercise 6

1. Discuss the motion of a particle that is placed on the inner surface of a spherical pot, close to its bottom, and released from hold (no friction).
2. Discuss the motion of a ball with radius $r$ that is placed on the inner surface of a spherical pot with radius $R$, assume $R >> r$, close to its bottom, and released from hold (enough friction).